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Soret and chemical reaction effects on a three-dimensional MHD convective flow of dissipative fluid along an infinite vertical porous plate

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Abstract

An analytical study was performed to study effects of thermo-diffusion and chemical reactions on a three-dimensional MHD mixed convective flow of dissipative fluid along an infinite vertical porous plate with transverse sinusoidal suction velocity. The parabolic partial differential equations governing the fluid flow, heat transfer, and mass transfer were solved using perturbation technique and the expressions for velocity, temperature, and concentration distributions were obtained. Expressions for skin friction at the plate in the direction of the main flow, rate of heat transfer, and mass transfer from the plate to the fluid were derived in a non-dimensional form. Velocity, temperature, concentration, amplitudes of the perturbed parts of skin friction, rate of heat transfer, rate of mass transfer, and skin friction at the plate are presented in graphs and effects of various physical parameters like Hartmann number M, Prandtl number Pr, Reynolds number Re, Schmidt number Sc, Soret number So, Grashof number for heat transfer Gr, Grashof number for mass transfer Gm, and chemical reaction parameter Kr on the above flow quantities were analyzed and then the obtained results were physically interpreted.

Nomenclature

$\vec{B}$	Magnetic induction vector
B_0	Iintensity of the applied magnetic field
C_p	Specific heat at constant pressure
$\overline{C}$	Species concentration
$\overline{C}_\infty$	Concentration of the fluid at infinity
$\overline{C}_w$	Concentration of the fluid at the plate
E	Eckert number

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$\vec{E}_0$	Electric filed
D	Coefficient of chemical molecular Mass diffusivity
Gr	Grashof number for heat transfer
Gm	Grashof number for mass transfer
g	Acceleration due to gravity
$\hat{i}, \hat{j}, \hat{k}$	Unit vectors along the co-ordinate axes
$\vec{J} \times \vec{B}$	Lorentz force per unit volume
$\frac{\vec{J}^2}{\sigma}$	Ohmic dissipation per unit volume
k	Thermal conductivity
L	Wave length of the periodic Suction velocity
M	Hartmann number
Pr	Prandtl number
P	pressure
p_∞	Gravitational pressure
$\vec{q}$	Velocity vector
Re	Reynolds number
Sc	Schmidt number
So	Soret number
Kr	Chemical reaction
$\overline{T}$	Temperature
$\overline{T}_\infty$	Temperature of the fluid at infinity
$\overline{T}_w$	Temperature of the fluid at the plate
$\overline{U}$	Free stream velocity
$(\overline{u}, \overline{v}, \overline{w})$	Components of $\vec{q}$
u	Dimensionless velocity in x-direction
v	Dimensionless velocity in y-direction
$\overline{v}_w$	Suction velocity
V_0	Mean suction velocity
w	Dimensionless velocity in Z – direction
$(\overline{x}, \overline{y}, \overline{z})$	Cartesian coordinates
x, y, z	Dimensionless coordinates Perpendicular to the free stream velocity

Greek symbols

α	Thermal diffusivity
β	Co-efficient of volume expansion for Thermal expansion
$\overline{\beta}$	The volumetric co-efficient of Expansion with concentration
ε	Small reference parameter ($\varepsilon << 1$)
ϕ	Dimensionless concentration
θ	Dimensionless temperature
ρ	Density of the fluid
ρ_∞	Density of the fluid in the free stream
σ	Electric conductivity
υ	Kinematic viscosity

1. Introduction

Many natural phenomena and technological problems are susceptible to MHD analysis. Geophysics encounters MHD characteristics in the interactions of conducting fluids and magnetic fields. Engineers employ MHD principle in the design of heat exchangers, pumps, and flow meters, in space vehicle propulsion, thermal protection, braking, control, and re-entry, in creating novel power generating systems, etc. From technological point of view, MHD convection flow problems are also very significant in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering, and electronics. Model studies of the above phenomena of MHD convection have been done by Ferraro and Plumpton [1], Cramer and Pai [2], Sanyal and Bhattacharya [3], and Nikodijevic et al. [4].

On the other hand, along with free convection currents caused by temperature difference, flow is also affected by the difference in concentrations on material constitutions. Many investigators (such as Raptis and Kafoussias [5], Bejan and Khair [6], Singh and Singh [7], Acharya et al. [8], Babu and Rao [9], and Singh et al. [10]) have studied the phenomena of MHD free convection and mass transfer flow. Problems of laminar flow control have been investigated by many researchers owing to its importance in the field of aeronautical engineering considering its application in substantially reducing drag and hence the vehicle power requirement. Development of this subject has been compiled by Lachman [11].

Theoretical and experimental investigations have indicated that transition from laminar to turbulent flow which causes the drag coefficient to increase may be prevented by the fluid suction through the application of transverse magnetic field and by heat and mass transfer from the boundary layer to the wall. To obtain any desired reduction in the drag, increasing suction alone is uneconomical since energy consumptions of the suction pumps will be higher. Therefore the method of "cooling of the wall" in controlling laminar flow together with the application of suction has become more useful and hence received the attention of many workers.

Effect of the flow caused by the periodic suction velocity perpendicular to the main flow, when the difference between the wall temperature and free stream temperature gives rise to buoyancy force in the direction of the free stream on heat transfer characteristics, was investigated by Singh et al. [12], which was extended by Ahmed and Sharma [13] to the case when the medium was porous. Gupta and Johari [14] analyzed the effects of magnetic field on the three-dimensional forced flow of an incompressible viscous fluid past a porous plate. Singh and Sharma [15] studied the effect of the porosity of the porous medium on the three-dimensional couette flow and heat transfer. Ahmed et al. [16] obtained an analytical solution for the problem of the three-dimensional free convective flow of an incompressible viscous fluid past a porous vertical plate with the transverse sinusoidal suction velocity considering the presence of species concentration.

Thermal-diffusion (Soret) effect, for instance, was utilized for isotope separation and, in the mixer between gases with very light (H_2, H_e) and medium (N_2, air) molecular weight, the diffusion-thermo (Dufour) effect was found to be in the order of considerable magnitude such that it

cannot be ignored according to Eckert and Drake [17]. Taking into account the importance of the above-mentioned effects, Kafoussias and Williams [18] studied thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature-dependent viscosity. Postelnicu [19] numerically investigated the influence of a magnetic field on heat and mass transfer by natural convection for vertical surfaces in porous media considering Soret and Dufour effects.

Seungsoo et al. [20] developed a complete three-dimensional mathematical model governing the steady, laminar flow of an incompressible fluid subjected to a magnetic field including internal heating due to the Joule effect, heat transfer due to conduction, and thermally induced buoyancy forces. Results of test cases with thermally induced buoyancy demonstrated stabilizing effect of the magnetic field on the re-circulating flows. However, in convective heat and mass transfer process, diffusion rates can be tremendously altered by chemical reaction. Effect of a chemical reaction depends on whether the reaction is heterogeneous or homogeneous. It also depends on whether the reaction occurs at an interface or as a single-phase volume reaction. A reaction is said to be of the order *n* if the reaction rate is proportional to the n^{th} power of the concentration. In particular, a reaction is said to be the first order if its rate is directly proportional to the concentration itself. In nature, the presence of pure air or water is not possible. Some foreign mass may be present either naturally or mixed with the air or water which causes some kind of chemical reaction. Studying such a type of chemical reaction processes is useful for improving a number of chemical technologies, such as food processing and polymer production. Effects of mass transfer on moving isothermal vertical plate in the presence of chemical reaction were studied by Das et al. [21] Muthucumaraswamy et al. [22] studied the effect of chemical reaction on unsteady MHD flow through an impulsively started semi-infinite vertical plate. Effects of Dufour and Soret on steady free convection and mass transfer flow past a semi-infinite vertical porous plate in a porous medium were studied by Alam et al. [23]. Anjali Devi and Wilfred Samuel Raj [24] have investigated thermo-diffusion effects on unsteady hydromagnetic free convection flow with heat and mass transfer past a moving vertical plate with time-dependent suction and heat source in a slip flow regime.

Ahmed [25] studied effect of the magnetic field on a three-dimensional mixed convective flow

with mass transfer along an infinite vertical porous plate. To the best knowledge of the current authors, no attempts have been made for studying the effect of a transverse magnetic field on a mixed convective flow of an incompressible viscous, electrically conducting fluid with mass transfer along a vertical porous plate with transverse sinusoidal velocity considering the effect of ohmic and viscous dissipations together. The above-explained attempt was made in the present work.
In this paper, an analytical study was performed to investigate the effects of thermo-diffusion and chemical reactions on the three-dimensional MHD mixed convective flow of dissipative fluid along an infinite vertical porous plate with transverse sinusoidal suction velocity.

2. Mathematical formulation

Consider the steady free and forced convection flow of an incompressible viscous electrically conducting fluid taking into account the species concentration past a vertical porous plate with transverse sinusoidal suction velocity in the presence of Thermo-diffusion and chemical reaction effects. We make the following assumptions.

i. All the fluid properties except the density in the buoyancy force term are constants.
ii. A magnetic field of uniform strength B_0 is applied transversely to the direction of the free stream.
iii. The magnetic Reynolds number Re is small so that the induced Magnetic field can be neglected.
iv. $\overline{T}_w > \overline{T}_\infty$ and $\overline{C}_w > \overline{C}_\infty$

Introducing a coordinate system $(\overline{x}, \overline{y}, \overline{z})$ with X-axis vertically upwards along the plate, Y-axis perpendicular to it and directed into the fluid region and Z-axis along the width of the plate. Let $q = \hat{i}\overline{u} + \hat{j}\overline{v} + \hat{k}\overline{w}$ be the fluid velocity at the point $(\overline{x}, \overline{y}, \overline{z})$ and $\overline{B} = B_0\hat{j}$ be the applied magnetic field. The transverse sinusoidal suction velocity is given by

$$\overline{v}_w(\overline{z}) = -V_0\left[1 + \varepsilon\cos\frac{\pi\overline{z}}{L}\right] \tag{1}$$

This consists of basic steady distribution V_0 with superimposed weak distribution $\varepsilon V_0 \cos\frac{\pi z}{L}$ confined in the boundary layer only. Here negative sign indicates that the direction of the suction velocity is towards the plate. This suction velocity $\overline{v}_w(\overline{z})$ is applied transversely to the plate and weak distribution $\varepsilon V_0 \cos\frac{\pi z}{L}$ will have no role in the outer edge of the boundary layer.
Due to the application of suction at the surface, the fluid particles at the edge of the boundary layer will have a tendency to get displaced towards the plate surface. Therefore $\overline{v} \to -V_0$ at $\overline{y} \to \infty$. This phenomenon is clearly supported by the equation of continuity. The velocity, temperature and concentration fields are independent of $\overline{x}$, because an asymptotic flow has been considered but the flow itself is three dimensional due to cross flow.

With the foregoing assumptions, following are the equations governing the flow

A. Continuity equation

$$\frac{\partial\overline{v}}{\partial\overline{y}} + \frac{\partial\overline{w}}{\partial\overline{z}} = 0 \tag{2}$$

X-component of momentum equation

$$\rho\left(\overline{v}\frac{\partial\overline{u}}{\partial\overline{y}} + \overline{w}\frac{\partial\overline{u}}{\partial\overline{z}}\right) = -\frac{\partial p}{\partial\overline{x}} - \rho g + \mu\left(\frac{\partial^2\overline{u}}{\partial\overline{y}^2} + \frac{\partial^2\overline{u}}{\partial\overline{z}^2}\right) - \sigma B_0{}^2\overline{u} \tag{3}$$

At the outer edge of the boundary layer the parallel component $\overline{u} = \overline{U}$, the free stream velocity. Since there is no large velocity gradient here, the viscous term in the equation

(3) vanishes for small μand hence for the outer flow, we have

$$0 = -\frac{\partial p_\infty}{\partial \overline{x}} - \rho_\infty g - \sigma B_0^{\,2}\overline{U} \quad (4)$$

It is emphasized by Schlichting [26] that in case of hot vertical plate, the pressure in each horizontal plane is equal to the gravitational pressure. That is p = p_∞. Hence (4) reduces to

$$0 = -\frac{\partial p}{\partial \overline{x}} - \rho_\infty g - \sigma B_0^{\,2}\overline{U} \quad (5)$$

By eliminating the pressure term from the Eqs. (3 and 5), we obtain

$$\rho\left(\overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} + \overline{w}\frac{\partial \overline{u}}{\partial \overline{z}}\right) = (\rho_\infty - \rho)g + \mu\left(\frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + \frac{\partial^2 \overline{u}}{\partial \overline{z}^2}\right) + \sigma B_0^{\,2}(\overline{U} - \overline{u}) \quad (6)$$

The Boussinesq approximation gives

$$\rho_\infty - \rho = \rho_\infty \beta(\overline{T} - \overline{T}_\infty) + \rho_\infty \overline{\beta}(\overline{C} - \overline{C}_\infty) \quad (7)$$

On using Eq. (7) in (6) and noting that ρ_∞ is approximately equal to ρ, we obtain the momentum equations as follows

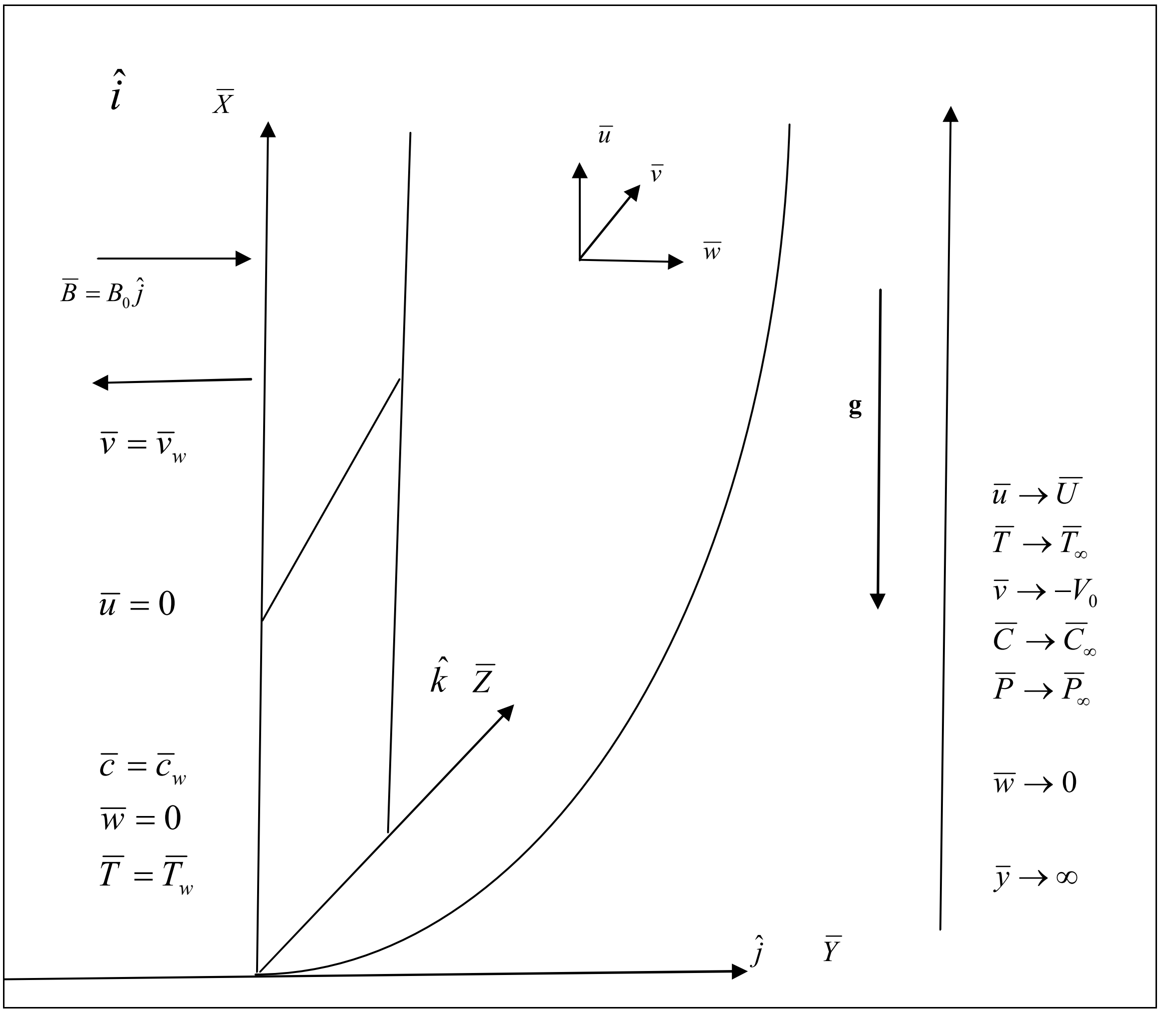

Fig. 1. The flow configuration.

B. $\bar{x}$ - component

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{u}}{\partial \bar{z}}=g\beta(\bar{T}-\bar{T}_\infty)+g\bar{\beta}(\bar{C}-\bar{C}_\infty)$$
$$+\upsilon\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right)+\frac{\sigma B_0^2}{\rho}(\bar{U}-\bar{u}) \tag{8}$$

C. $\bar{y}$ - component

$$\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{v}}{\partial \bar{z}}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}}+\upsilon\left(\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\right) \tag{9}$$

D. $\bar{z}$ - component

$$\bar{v}\frac{\partial \bar{w}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{w}}{\partial \bar{z}}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}}$$
$$+\upsilon\left(\frac{\partial^2 \bar{w}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right)-\frac{\sigma B_0^2 \bar{w}}{\rho} \tag{10}$$

E. Energy equation

$$\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{T}}{\partial \bar{z}}=\alpha\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right)$$
$$+\frac{\sigma B_0^2}{\rho C_p}\left[(\bar{U}-\bar{u})^2+\bar{w}^2\right]$$
$$+\frac{\upsilon}{C_P}\left[\begin{array}{l}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2+\left(\frac{\partial \bar{u}}{\partial \bar{z}}\right)^2+\left(\frac{\partial \bar{v}}{\partial \bar{z}}+\frac{\partial \bar{w}}{\partial \bar{y}}\right)^2\\+2\left\{\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2+\left(\frac{\partial \bar{w}}{\partial \bar{z}}\right)^2\right\}\end{array}\right] \tag{11}$$

F. Mass transfer equation

$$\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{C}}{\partial \bar{z}}=D\left(\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right)$$
$$+D_1\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right)+D_2(\bar{C}-\bar{C}_\infty) \tag{12}$$

The symbols involved have been defined in the nomenclature. The corresponding boundary conditions are given by

$$\bar{y}=0: \quad \bar{u}=0,\quad \bar{v}=\bar{v}_w,\quad \bar{w}=0,$$
$$\bar{T}=\bar{T}_w,\quad \bar{C}=\bar{C}_w$$
$$\bar{y}\to\infty: \quad \bar{u}=U,\quad \bar{v}=-V_0,\quad \bar{w}=0,$$
$$\bar{T}=\bar{T}_\infty,\quad \bar{C}=\bar{C}_\infty,\quad \bar{P}=\bar{P}_\infty$$

(13)

The following dimensionless quantities are introduced into the Eqs. (2, 8, 9, 10, 11 and 12), the governing equations become

$$y=\frac{\bar{y}}{L}, z=\frac{\bar{z}}{L}, u=\frac{\bar{u}}{V_0}, v=\frac{\bar{v}}{V_0}, U=\frac{\bar{U}}{V_0},$$
$$w=\frac{\bar{w}}{V_0}, Sc=\frac{\upsilon}{D},\ Kr=\frac{D_2L^2}{\upsilon}, \theta=\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_W-\bar{T}_\infty},$$
$$\phi=\frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty}, \Pr=\frac{\upsilon}{\alpha}, So=\frac{D_1(\bar{T}_w-\bar{T}_\infty)}{(\bar{C}_w-\bar{C}_\infty)},$$
$$Gr=\frac{Lg\beta(\bar{T}_w-\bar{T}_\infty)}{V_0^2}, p=\frac{\bar{p}}{\rho\left(\frac{\upsilon}{L}\right)^2},$$
$$Gm=\frac{Lg\bar{\beta}(\bar{C}_w-\bar{C}_\infty)}{V_0^2}, E=\frac{V_0^2}{C_p(\bar{T}_w-\bar{T}_\infty)},$$
$$M=\frac{\sigma B_0^2\upsilon}{\rho V_0^2}, \mathrm{Re}=\frac{V_0L}{\upsilon}, P_\infty=\frac{\bar{p}_\infty}{\rho\left(\frac{\upsilon}{L}\right)^2} \tag{14}$$

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{15}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=Gr\theta+Gm\phi$$
$$+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+M\,\mathrm{Re}(U-u) \tag{16}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\mathrm{Re}^2}\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{17}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\mathrm{Re}^2}\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-M\,\mathrm{Re}\,w \tag{18}$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{\Pr \mathrm{Re}}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)+\frac{E}{\mathrm{Re}}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right]+\frac{2E}{\mathrm{Re}}\left[\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\frac{E}{\mathrm{Re}}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2+ME\,\mathrm{Re}\left\{(U-u)^2+w^2\right\} \tag{19}$$

$$v\frac{\partial \phi}{\partial y}+w\frac{\partial \phi}{\partial z}=\frac{1}{\mathrm{Re}}\left\{\frac{1}{Sc}\left(\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}\right)+So\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)+Kr\phi\right\} \tag{20}$$

The relevant boundary conditions obtained from (13) using the dimensionless quantities (14), are given as

$$\left.\begin{array}{l} y=0 \quad : u=0, v=-(1+\varepsilon Cos\pi z), \\ \qquad\quad w=0, \theta=1, \phi=1 \\ y\to\infty : u=U, v=-1, w=0, \\ \qquad\quad \theta=0, \phi=0, p=p_\infty \end{array}\right\} \tag{21}$$

3. Method of solution

We assume the solutions of the Eqs. (15-20) to be of the form

$u = u_0(y) + \varepsilon\, u_1(y, z) + O(\varepsilon^2)$ (22)

$v = v_0(y) + \varepsilon\, v_1(y, z) + O(\varepsilon^2)$ (23)

$w = w_0(y) + \varepsilon\, w_1(y, z) + O(\varepsilon^2)$ (24)

$p = p_0(y) + \varepsilon\, p_1(y, z) + O(\varepsilon^2)$ (25)

$\theta = \theta_0(y) + \varepsilon\, \theta_1(y, z) + O(\varepsilon^2)$ (26)

$\phi = \phi_o(y) + \varepsilon\, \phi_1(y, z) + O(\varepsilon^2)$ (27)

with $p_0 = p_\infty$, $w_0 = 0$

Substituting these in the Eqs. (15-20) and equating the co-efficient of same degree terms and neglecting ε^2, we get the following sets of the differential equations.

(a) Zeroth - order equations

$$\frac{dv_0}{dy}=0 \tag{28}$$

$$v_0\frac{du_0}{dy}=Gr\theta_0+Gm\phi_0+\frac{1}{\mathrm{Re}}\frac{d^2u_0}{dy^2}+M\,\mathrm{Re}(U-u_0) \tag{29}$$

$$v_0\frac{d\theta_0}{dy}=\frac{1}{\Pr \mathrm{Re}}\frac{d^2\theta_0}{dy^2}+\frac{2E}{\mathrm{Re}}\left(\frac{dv_0}{dy}\right)^2+\frac{E}{\mathrm{Re}}\left(\frac{du_0}{dy}\right)^2+ME\,\mathrm{Re}(U-u_0)^2 \tag{30}$$

$$v_0\frac{d\phi_0}{dy}=\frac{1}{\mathrm{Re}}\left[\frac{1}{Sc}\frac{d^2\phi_0}{dy^2}+So\frac{d^2\theta_0}{dy^2}+Kr\phi_0\right] \tag{31}$$

(b) First order equations

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0 \tag{32}$$

$$-\frac{\partial u_1}{\partial y}+v_1\frac{\partial u_0}{\partial y}=Gr\theta_1+Gm\phi_1+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}\right)-M\,\mathrm{Re}\,u_1 \tag{33}$$

$$-\frac{\partial v_1}{\partial y}=-\frac{1}{\mathrm{Re}^2}\frac{\partial p_1}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}\right) \tag{34}$$

$$-\frac{\partial w_1}{\partial y}=-\frac{1}{\mathrm{Re}^2}\frac{\partial p_1}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}\right)-M\,\mathrm{Re}\,w_1 \tag{35}$$

$$-\frac{\partial \theta_1}{\partial y}+v_1\frac{d\theta_0}{dy}=\frac{1}{\mathrm{Pr}\,\mathrm{Re}}\left(\frac{\partial^2 \theta_1}{\partial y^2}+\frac{\partial^2 \theta_1}{\partial z^2}\right)+\frac{2E}{\mathrm{Re}}\frac{du_0}{dy}\frac{du_1}{dy}+\frac{4E}{\mathrm{Re}}\frac{dv_0}{dy}\frac{\partial v_1}{\partial y}-2M\,\mathrm{Re}(U-u_0)u_1 \tag{36}$$

$$-\frac{\partial \phi_1}{\partial y}+v_1\frac{d\phi_0}{dy}=\frac{1}{\mathrm{Re}}\left\{\begin{array}{l}\frac{1}{Sc}\left(\frac{\partial^2 \phi_1}{\partial y^2}+\frac{\partial^2 \phi_1}{\partial z^2}\right)\\ +So\left(\frac{\partial^2 \theta_1}{\partial y^2}+\frac{\partial^2 \theta_1}{\partial z^2}\right)\\ +Kr\phi_1\end{array}\right\} \tag{37}$$

with conditions

$$\left.\begin{array}{l}y=0\ :u_0=0,v_0=-1,\theta_0=1,\\ \qquad \phi_0=1,u_1=0,v_1=-\cos\pi z,\\ \qquad w_1=0,\theta_1=0,\phi_1=0\\ y\to\infty:u_0=U,v_0=-1,\theta_0=0,\\ \qquad \phi_0=0,u_1=0,v_1=0,w_1=0,\\ \qquad p_1=0,\theta_1=0,\phi_1=0\end{array}\right\} \tag{38}$$

The solutions of the Eq. (28) subject to boundary conditions (Eq. (38)) are respectively

$$v_0=-1 \tag{39}$$

In order to solve the Eqs. (29-31) under the above boundary conditions, we note that E<1 for all the incompressible fluids and it is assumed that the solutions of these equations to be of the form

$$u_0(y)=u_{00}(y)+Eu_{01}(y)+O(E^2) \tag{40}$$

$$\theta_0(y)=\theta_{00}(y)+E\theta_{01}(y)+O(E^2) \tag{41}$$

$$\phi_0(y)=\phi_{00}(y)+E\phi_{01}(y)+O(E^2) \tag{42}$$

Substituting from Eqs. (40-42) in the Eqs. (29-31) and equating the co-efficient of the same degree terms and neglecting the term of $O(E^2)$, the following differential equations with corresponding boundary conditions are derived.

$$u''_{00}+\mathrm{Re}u'_{00}-M\,\mathrm{Re}^2u_{00}=-M\,\mathrm{Re}^2U-Gr\mathrm{Re}\theta_{00}-Gm\mathrm{Re}\phi_{00} \tag{43}$$

$$u''_{01}+\mathrm{Re}u'_{01}-M\,\mathrm{Re}^2u_{01}=-Gr\mathrm{Re}\theta_{01}-Gm\mathrm{Re}\phi_{01} \tag{44}$$

$$\theta''_{00}+\mathrm{Pr}\,\mathrm{Re}\theta'_{00}=0 \tag{45}$$

$$\theta''_{01}+\mathrm{Pr}\,\mathrm{Re}\theta'_{01}=-\mathrm{Pr}u'^2_{00}-M\,\mathrm{Pr}\,\mathrm{Re}^2(U-u_{00})^2 \tag{46}$$

$$\phi''_{00}+\mathrm{Re}\,Sc\phi'_{00}-ScKr\phi_{00}=-ScSo\theta''_{00} \tag{47}$$

$$\phi_{01}'' + \mathrm{Re}\,Sc\phi_{01}' + ScKr\phi_{01} = -ScSo\theta_{01}'' \quad (48)$$

subject to the boundary conditions

$$\left.\begin{aligned} y=0 \ &: u_{00}=0,\ u_{01}=0, \theta_{00}=1, \\ &\theta_{01}=0, \phi_{00}=1, \phi_{01}=0 \\ y\to\infty &: u_{00}=U, u_{01}=0, \theta_{00}=0, \\ &\theta_{01}=0, \phi_{00}=0, \phi_{01}=0 \end{aligned}\right\} \quad (49)$$

The solutions of these Eqs. (43-48) under the boundary conditions (Eq. (49)) are as follows

$$\theta_{00} = e^{-t_1 y} \quad (50)$$

$$\phi_{00} = C_0 e^{-t_0 y} + C_1 e^{-t_1 y} \quad (51)$$

$$u_{00} = U + C_2 e^{-t_2 y} + C_3 e^{-t_1 y} + C_4 e^{-t_0 y} \quad (52)$$

$$\begin{aligned} \theta_{01} &= C_5 e^{-t_1 y} + C_6 e^{-2t_0 y} + C_7 e^{-2t_1 y} \\ &+ C_8 e^{-2t_2 y} + C_9 e^{-t_3 y} + C_{10} e^{-t_4 y} + C_{11} e^{-t_5 y} \end{aligned} \quad (53)$$

$$\begin{aligned} \phi_{01} &= C_{12} e^{-t_6 y} + C_{13} e^{-t_1 y} + C_{14} e^{-2t_0 y} \\ &+ C_{15} e^{-2t_1 y} + C_{16} e^{-2t_2 y} + C_{17} e^{-t_3 y} \\ &+ C_{18} e^{-t_4 y} + C_{19} e^{-t_5 y} \end{aligned} \quad (54)$$

$$\begin{aligned} u_{01} &= C_{20} e^{-t_2 y} + C_{21} e^{-t_1 y} + C_{22} e^{-2t_0 y} \\ &+ C_{23} e^{-2t_1 y} + C_{24} e^{-2t_2 y} + C_{25} e^{-t_3 y} \\ &+ C_{26} e^{-t_4 y} + C_{27} e^{-t_5 y} + C_{28} e^{-t_6 y} \end{aligned} \quad (55)$$

(c) Cross flow solution

We shall first consider the Eqs. (32, 34 and 35) for $v_1(y, z)$, $w_1(y, z)$, $p_1(y, z)$ which are independent of main flow component u_1, temperature field θ_1 and concentration field ϕ_1.

The suction velocity $v_w = -(1+ \varepsilon \cos\pi z)$ consists of a basic uniform distribution with superimposed weak sinusoidal distribution $\varepsilon \cos\pi z$. Hence, the velocity components v, w and p are also separated into mean and small sinusoidal components v_1, w_1, p_1. We assume v_1, w_1 and p_1 to be of the following form:

$$\left.\begin{aligned} v_1 &= -\pi\, v_{11}(y)\cos\pi z \\ w_1 &= v_{11}'(y)\sin\pi z \\ p_1 &= \mathrm{Re}^2\, p_{11}(y)\cos\pi z \end{aligned}\right\} \quad (56)$$

On substitution of Eq. (56) the Eq. (32) is satisfied and the Eqs. (34 and 35) reduce to the following differential equations

$$v_{11}'' + \mathrm{Re}\, v_{11}' - \pi^2 v_{11} = -\frac{\mathrm{Re}\, p_{11}'}{\pi} \quad (57)$$

$$\begin{aligned} v_{11}''' + \mathrm{Re}\, v_{11}'' - (\pi^2 + M\,\mathrm{Re}^2) v_{11}' = \\ -\mathrm{Re}\,\pi\, p_{11} \end{aligned} \quad (58)$$

The relevant boundary conditions for these equations are

$$\left.\begin{aligned} y=0 \ &: v_{11} = \frac{1}{\pi}, v_{11}' = 0 \\ y\to\infty &: v_{11} = 0, v_{11}' = 0 \end{aligned}\right\} \quad (59)$$

he solutions of these Eqs. (57 and 58) subject to the boundary conditions (Eq. (59)) is

$$v_{11} = \frac{1}{\pi(\bar{\xi} - \xi)}\left[\bar{\xi} e^{-\xi y} - \xi e^{-\bar{\xi} y}\right] \quad (60)$$

Hence,the solution for the velocity components v_1, w_1 and pressure p_1 are as follows

$$v_1 = \frac{1}{(\xi - \bar{\xi})}\left[\bar{\xi} e^{-\xi y} - \xi e^{-\bar{\xi} y}\right]\cos\pi z, \quad (61)$$

$$w_1 = \frac{\xi\bar{\xi}}{\pi(\bar{\xi} - \xi)}\left[e^{-\bar{\xi} y} - e^{-\xi y}\right]\sin\pi z \quad (62)$$

$$p_1 = \frac{\mathrm{Re}\,\xi\bar{\xi}}{\pi^2(\bar{\xi} - \xi)}\left[\bar{\xi}_1 e^{-\bar{\xi} y} - \xi_1 e^{-\xi y}\right]\cos\pi z \quad (63)$$

where

$$\xi_1 = g + \mathrm{Re}\,\xi - \xi^2, \quad \bar{\xi}_1 = g + \mathrm{Re}\,\bar{\xi} - \bar{\xi}^2,$$

$$g = \pi^2 + M\,\mathrm{Re}^2$$

Solutions of the main flow temperature and species concentration fields

We shall now consider the Eqs. (33, 36 and 37). The solutions for the velocity component u, concentration field ϕ and temperature field θ are also separated into mean and sinusoidal components u_1, θ_1, ϕ_1.

$$\left.\begin{aligned} u_1 &= u_{11}(y)\cos\pi z \\ \theta_1 &= \theta_{11}(y)\cos\pi z \\ \phi_1 &= \phi_{11}(y)\cos\pi z \end{aligned}\right\} \tag{64}$$

Using the expressions for v_1, u_1, θ_1, ϕ_1 in Eqs. (33, 36, 37) we get the following ordinary differential equations

$$u_{11}'' + \mathrm{Re}\,u_{11}' - g u_{11} = -\pi\,\mathrm{Re}\,v_{11}u_0' - d\theta_{11} - e\phi_{11} \tag{65}$$

$$\begin{aligned} \theta_{11}'' + t_1\theta_{11}' - \pi^2\theta_{11} = &-\pi\, t_1 v_{11}\theta_0' - 2E\,\mathrm{Pr}\,u_0' u_{11}' \\ &+ 2EMt_1\,\mathrm{Re}(U - u_0)u_{11} \end{aligned} \tag{66}$$

$$\begin{aligned} \phi_{11}'' + b\phi_{11}' + (c - \pi^2)\varphi_{11} = &-\pi b v_{11}\phi_0^1 \\ &- f(\theta_{11}'' - \pi^2\theta_{11}) \end{aligned} \tag{67}$$

with the boundary conditions

$$\left.\begin{aligned} y = 0 \ &: u_{11} = 0, \theta_{11} = 1, \phi_{11} = 0 \\ y \to \infty &: u_{11} = 0, \theta_{11} = 1, \phi_{11} = 0 \end{aligned}\right\} \tag{68}$$

Now in order to solve the Eqs. (65-67), the solutions of these three equations are assumed to be of the form

$$\left.\begin{aligned} u_{11}(y) &= f_0(y) + E\, f_1(y) + O(E^2) \\ \theta_{11}(y) &= g_0(y) + E\, g_1(y) + O(E^2) \\ \phi_{11}(y) &= \psi_0(y) + E\, \psi_1(y) + O(E^2) \end{aligned}\right\} \tag{69}$$

Substituting these in the Eqs. (65-67) and equating the coefficients of similar terms and neglects E^2, we get the following differential equations:

$$\begin{aligned} f_0'' + \mathrm{Re}\,f_0' - g\, f_0 = &-\pi\,\mathrm{Re}\,v_{11}u_{00}' \\ &- dg_0 - e\psi_0 \end{aligned} \tag{70}$$

$$\begin{aligned} f_1'' + \mathrm{Re}\,f_1' - g\, f_1 = &-\pi\,\mathrm{Re}\,v_{11}u_{01}' \\ &- dg_1 - e\psi_1 \end{aligned} \tag{71}$$

$$g_0'' + t_1 g_0' - \pi^2 g_0 = -\pi\, t_1 v_{11}\theta_{00}' \tag{72}$$

$$\begin{aligned} &g_1'' + t_1 g_1' - \pi^2 g_1 = -\pi\, t_1 v_{11}\theta_{01}' \\ &- 2\,\mathrm{Pr}\,u_{00}' f_0' + 2Mt_1\,\mathrm{Re}\, f_0(U - u_{00}) \end{aligned} \tag{73}$$

$$\begin{aligned} \psi_0'' + b\psi_0' + (c - \pi^2)\psi_0 = &-\pi b v_{11}\phi_{00}' \\ &- f\, g_0'' + f\,\pi^2 g_0 \end{aligned} \tag{74}$$

$$\begin{aligned} \psi_1'' + b\psi_1' + (c - \pi^2)\psi_1 = &-\pi b v_{11}\phi_{01}' \\ &- f\, g_1'' + f\,\pi^2 g_1 \end{aligned} \tag{75}$$

with boundary conditions

$$\left.\begin{aligned} y = 0 \ &: f_0 = 0,\ f_1 = 0, g_0 = 0,\ g_1 = 0, \\ &\quad \psi_0 = 0, \psi_1 = 0 \\ y \to \infty &: f_0 = 0,\ f_1 = 0, g_0 = 0,\ g_1 = 0, \\ &\quad \psi_0 = 0, \psi_1 = 0 \end{aligned}\right\} \tag{76}$$

The solutions of these equations under the boundary conditions (Eq. (76)) are as follows

$$g_0 = C_{29}e^{-t_7 y} + C_{30}e^{-t_8 y} + C_{31}e^{-t_9 y} \tag{77}$$

$$\begin{aligned} \psi_0 = &C_{32}e^{-t_{12} y} + C_{33}e^{-t_7 y} + C_{34}e^{-t_8 y} \\ &+ C_{35}e^{-t_9 y} + C_{36}e^{-t_{10} y} + C_{37}e^{-t_{11} y} \end{aligned} \tag{78}$$

$$f_0 = C_{38}e^{-t_{15}y} + C_{39}e^{-t_7 y} + C_{40}e^{-t_8 y} + C_{41}e^{-t_9 y} + C_{42}e^{-t_{10} y} + C_{43}e^{-t_{11} y} + C_{44}e^{-t_{12} y} + C_{45}e^{-t_{13} y} + C_{46}e^{-t_{14} y} \quad (79)$$

$$g_1 = C_{47}e^{-t_7 y} + C_{48}e^{-t_8 y} + C_{49}e^{-t_9 y} + C_{50}e^{-t_{16} y} + C_{51}e^{-t_{17} y} + C_{52}e^{-t_{18} y} + C_{53}e^{-t_{19} y} + C_{54}e^{-t_{20} y} + C_{55}e^{-t_{21} y} + C_{56}e^{-t_{22} y} + C_{57}e^{-t_{23} y} + C_{58}e^{-t_{24} y} + C_{59}e^{-t_{25} y} + C_{60}e^{-t_{26} y} + C_{61}e^{-t_{27} y} + C_{62}e^{-t_{28} y} + C_{63}e^{-t_{29} y} + C_{64}e^{-t_{30} y} + C_{65}e^{-t_{31} y} + C_{66}e^{-t_{32} y} + C_{67}e^{-t_{33} y} + C_{68}e^{-t_{34} y} + C_{69}e^{-t_{35} y} + C_{70}e^{-t_{36} y} + C_{71}e^{-t_{37} y} + C_{72}e^{-t_{38} y} + C_{73}e^{-t_{39} y} + C_{74}e^{-t_{40} y} + C_{75}e^{-t_{41} y} + C_{76}e^{-t_{42} y} + C_{77}e^{-t_{43} y} + C_{78}e^{-t_{44} y} + C_{79}e^{-t_{45} y} + C_{80}e^{-t_{46} y} + C_{81}e^{-t_{47} y} + C_{82}e^{-t_{48} y} + C_{83}e^{-t_{49} y} + C_{84}e^{-t_{50} y} + C_{85}e^{-t_{51} y} + C_{86}e^{-t_{52} y} + C_{87}e^{-t_{53} y} + C_{88}e^{-t_{54} y} \quad (80)$$

$$\psi_1 = C_{89}e^{-t_{12} y} + C_{90}e^{-t_{55} y} + C_{91}e^{-t_{56} y} + C_{92}e^{-t_7 y} + C_{93}e^{-t_8 y} + C_{94}e^{-t_9 y} + C_{95}e^{-t_{16} y} + C_{96}e^{-t_{17} y} + C_{97}e^{-t_{18} y} + C_{98}e^{-t_{19} y} + C_{99}e^{-t_{20} y} + C_{100}e^{-t_{21} y} + C_{101}e^{-t_{22} y} + C_{102}e^{-t_{23} y} + C_{103}e^{-t_{24} y} + C_{104}e^{-t_{25} y} + C_{105}e^{-t_{26} y} + C_{106}e^{-t_{27} y} + C_{107}e^{-t_{28} y} + C_{108}e^{-t_{29} y} + C_{109}e^{-t_{30} y} + C_{110}e^{-t_{31} y} + C_{111}e^{-t_{32} y} + C_{112}e^{-t_{33} y} + C_{113}e^{-t_{34} y} + C_{114}e^{-t_{35} y} + C_{115}e^{-t_{36} y} + C_{116}e^{-t_{37} y} + C_{117}e^{-t_{38} y} + C_{118}e^{-t_{39} y} + C_{119}e^{-t_{40} y} + C_{120}e^{-t_{41} y} + C_{121}e^{-t_{42} y} + C_{122}e^{-t_{43} y} + C_{123}e^{-t_{44} y} + C_{124}e^{-t_{45} y} + C_{125}e^{-t_{46} y} + C_{126}e^{-t_{47} y} + C_{127}e^{-t_{48} y} + C_{128}e^{-t_{49} y} + C_{129}e^{-t_{50} y} + C_{130}e^{-t_{51} y} + C_{131}e^{-t_{52} y} + C_{132}e^{-t_{53} y} + C_{133}e^{-t_{54} y} \quad (81)$$

$$f_1 = C_{134}e^{-t_{15} y} + C_{135}e^{-t_7 y} + C_{136}e^{-t_8 y} + C_{137}e^{-t_9 y} + C_{138}e^{-t_{13} y} + C_{139}e^{-t_{14} y} + C_{140}e^{-t_{16} y} + C_{141}e^{-t_{17} y} + C_{142}e^{-t_{18} y} + C_{143}e^{-t_{19} y} + C_{144}e^{-t_{20} y} + C_{145}e^{-t_{21} y} + C_{146}e^{-t_{22} y} + C_{147}e^{-t_{23} y} + C_{148}e^{-t_{24} y} + C_{149}e^{-t_{25} y} + C_{150}e^{-t_{26} y} + C_{151}e^{-t_{27} y} + C_{152}e^{-t_{28} y} + C_{153}e^{-t_{29} y} + C_{154}e^{-t_{30} y} + C_{155}e^{-t_{31} y} + C_{156}e^{-t_{32} y} + C_{157}e^{-t_{33} y} + C_{158}e^{-t_{34} y} + C_{159}e^{-t_{35} y} + C_{160}e^{-t_{36} y} + C_{161}e^{-t_{37} y} + C_{162}e^{-t_{38} y} + C_{163}e^{-t_{39} y} + C_{164}e^{-t_{40} y} + C_{165}e^{-t_{41} y} + C_{166}e^{-t_{42} y} + C_{167}e^{-t_{43} y} + C_{168}e^{-t_{44} y} + C_{169}e^{-t_{45} y} + C_{170}e^{-t_{46} y} + C_{171}e^{-t_{47} y} + C_{172}e^{-t_{48} y} + C_{173}e^{-t_{49} y} + C_{174}e^{-t_{50} y} + C_{175}e^{-t_{51} y} + C_{176}e^{-t_{52} y} + C_{177}e^{-t_{53} y} + C_{178}e^{-t_{54} y} + C_{179}e^{-t_{55} y} + C_{180}e^{-t_{56} y} \quad (82)$$

where C_0, C_1, C_2, ---,C_{180} are constants which are mentioned in the appendix.

Skin friction, Nusselt number and Sherwood number

The non-dimensional skin-friction in the direction of the free stream at the wall y = 0 is given by

$$\tau = -\frac{1}{\mathrm{Re}}\frac{\partial u}{\partial y}\bigg]_{y=0} = -\frac{1}{\mathrm{Re}}\left[u_0'(0) + \varepsilon u_{11}'(0)\cos\pi z\right] = \tau_0 + \varepsilon Q_1(\mathrm{Pr}, Sc, \mathrm{Re}, Gr, Gm, E, M)\cos\pi z \quad (83)$$

where

$$Q_1 = -\frac{u_{11}'(0)}{\mathrm{Re}}, \tau_0 = -\frac{u_0'(0)}{\mathrm{Re}}$$

The heat flux from the plate to the fluid in terms of Nusselt number is given by

$$Nu = -\frac{k}{\partial v_0 C_p (\bar{T}_w - \bar{T}_\infty)}\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{y=0}$$
$$= -\frac{1}{\Pr \mathrm{Re}}\frac{\partial \theta}{\partial y}\bigg]_{y=0}$$
$$= Nu_0 + \varepsilon Q_2(\Pr, \mathrm{Re}, Gr, Gm, E, M, Sc)\cos \pi z \quad (84)$$

where $Nu_0 = -\frac{1}{\Pr \mathrm{Re}}\theta_0^{'}(0),\ Q_2 = -\frac{\theta_{11}^{'}(0)}{\Pr \mathrm{Re}}$

The mass flux at the wall y = 0 in terms of Sherwood number *Sh* is given by

$$Sh = \frac{-1}{Sc\,\mathrm{Re}}\left(\frac{\partial \phi}{\partial y}\right)_{y=0}$$
$$= \frac{-1}{Sc\,\mathrm{Re}}\left[\phi_0^{'}(0) + \varepsilon \phi_{11}^{'}(0)\cos \pi z\right] \quad (85)$$
$$= 1 + \varepsilon Q_3(Sc, \mathrm{Re}, M)\cos \pi z$$

where $Q_3 = -\frac{\phi_{11}^{'}(0)}{Sc\,\mathrm{Re}}$

4. Results and discussion

In order to get the physical insight into the problem, the numerical values for u, θ, ϕ, τ, Q_1, Q_2 and Q_3 which are respectively the velocity, temperature, concentration, skin friction and amplitudes of the first order skin friction, first order Nusselt number and first order Sherwood number at the plate are obtained for different values of the physical parameters involved in the problem and these are demonstrated in graphs. The investigation is restricted to Prandtl number Pr = 7 which corresponds to water. The stream velocity U is taken to be equal to 1, E is selected to be 0.01 and z is taken to be equal to zero. The values of the other physical parameters namely M, Gr, Gm, Sc, So, Kr and Re are chosen arbitrarily.

From Fig. 2, it is observed that an increase in Hartmann number M leads to decrease in the velocity u. It is because that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing the velocity. It is inferred from Fig. 3, that an increase in chemical reaction parameter Kr results in a decrease in velocity u. This is due to the fact that hydrodynamics boundary layer becomes thin as the chemical reaction parameter increases. But this trend reverses completely for y > 8. Usually So accelerates the fluid velocity. It is observed that u increases as So increase in the region 0 ≤ y ≤ 4.5. But this trend reverses completely in the region very close to the moving plate as shown in the Fig. 4.

From Fig. 5 it is seen that, θ decreases as chemical reaction parameter Kr increases in the region 1 ≤ y ≤ 2. For y > 2 the complete reverse phenomenon is observed. near the plate and then decreases (for y > 1) as Soret number So increases. Fig. 6, shows that the temperature θ increases near the plate and then decreases (for y > 1) as Soret number So increases. The effects of Kr and So on concentration of the fluid ϕ are studied and the results are exhibited through Figs. 7 and 8. From Fig. 7, it is observed that near the plate ϕ increases with increasing Kr up to 0.5 and then decreases for further increasing in Kr. For 1 ≤ y ≤ 7, ϕ decreases as Kr increases. For 7 < y < 8.5, ϕ decreases with increasing Kr up to 0.5 and then increases for further increasing in Kr. For y > 8.5, the complete reverse phenomenon is observed. ϕ is increasing with increasing So. A complete reverse phenomenon is observed for y > 7 as shown in Fig. 8.

Figs. 9 and 10 depict the variation of the skin friction τ at the plate under the influences Re, So, Kr. From Fig. 9, it is noticed that an increase in chemical reaction parameter Kr results in a decrease in $|\tau|$. That is there is a reduction in the viscous drag (Shearing stress) on the plate due to the chemical reaction. It is observed from Fig. 10 that an increase in Soret number So leads to an increase in the magnitude of the skin friction $|\tau|$.

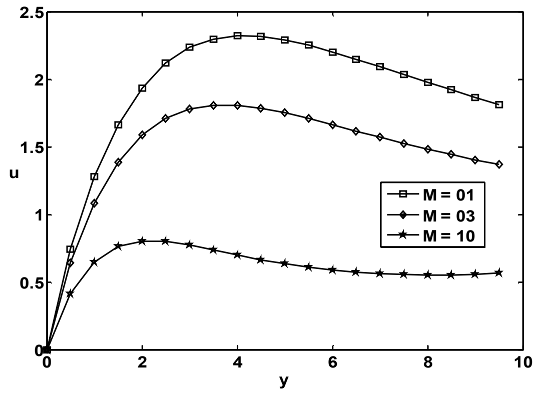

Fig. 2. Velocity profiles when Pr = 7, Sc = 1, ε = 0.01, So = 0.2, Gr =10, Gm = 10, Re = 0.05, Kr = 1.

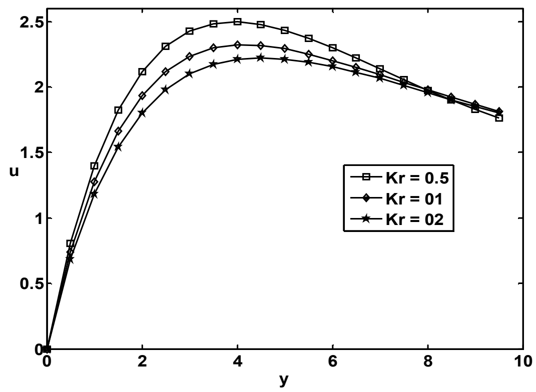

Fig. 3. Velocity profiles when Sc=1, Gr =10, ε = 0.01, So = 0.2, Gm = 10, Re = 0.05, M = 1, Pr = 7.

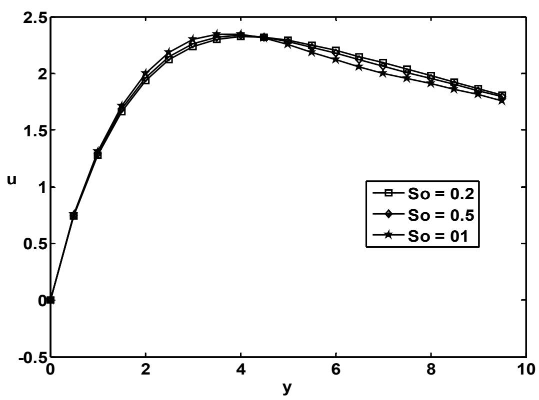

Fig. 4. Velocity profiles when Pr = 7, ε = 0.01, Gr = 10, Gm = 10, Re = 0.05, Kr =1, M =1, Sc =1.

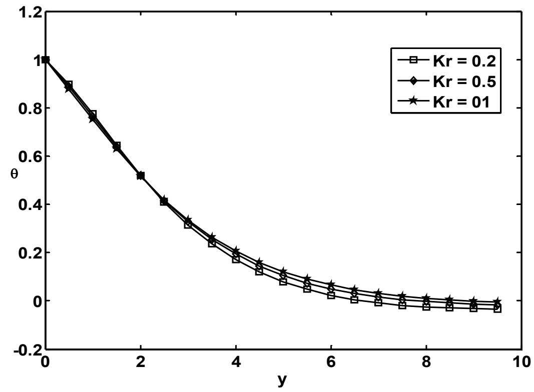

Fig. 5. Temperature profiles when Re = 0.05, Pr = 7, ε = 0.01, Gr = 10, Gm = 10, So = 0.2, M = 1, Sc = 1.

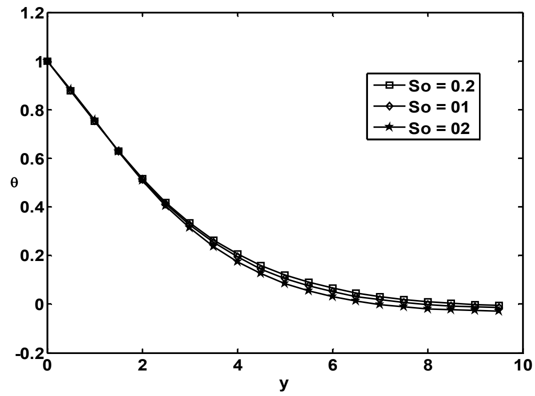

Fig. 6. Temperature profiles when Re = 0.05, Sc = 1, ε = 0.01, M = 1, Gm = 10, Kr = 1, Gr = 10, Pr = 7.

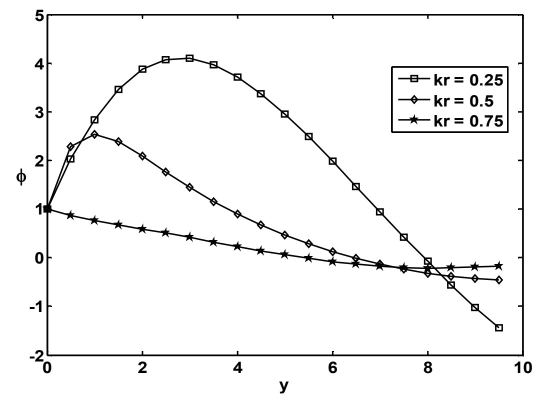

Fig. 7. Concentration profiles when Pr = 7, Gr = 10, Gm = 10, So = 0.2, Re = 0.05, M = 1, ε = 0.01, Sc = 1.

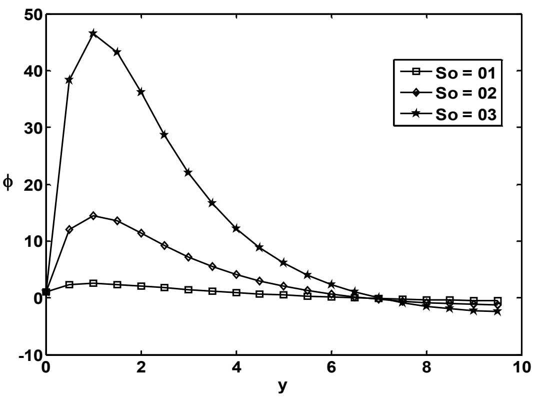

Fig. 8. Concentration profiles when Kr =1, ε = 0.01, Re = 0.05, Gr = 10, M = 1, Pr = 7, Gm = 10, Sc = 1.

This serves to accelerate the flow along the plate.

The change of behavior of Q_1, the amplitude of the first order skin friction at the plate against the Reynolds number Re under the effects of So, Kr and M is presented in Figs. 11 and 12. Fig. 11 shows that the Q_1 increases as M or So increases. Fig. 12 shows that Q_1 decreases as chemical reaction parameter Kr increases. The amplitude of skin friction (for z = 0) from the plate to the fluid drops due to increase of the chemical reaction. The variation of the amplitude Q_2 of the first order Nusselt number is demonstrated in Figs. 13 and 14. It is observed from Fig. 13 that Q_2 decreases for small values of Re ($0 \leq Re \leq 4$) and it becomes constant for $Re \geq 4$ as the chemical reaction parameter Kr increases. From Fig. 14, it is observed that Q_2 decreases with increasing So up to $Re \leq 4$. For $Re > 4$ the complete reverse phenomenon is observed.

The amplitude Q_3 of the first order Sherwood number are shown in Figs. 15 and 16.

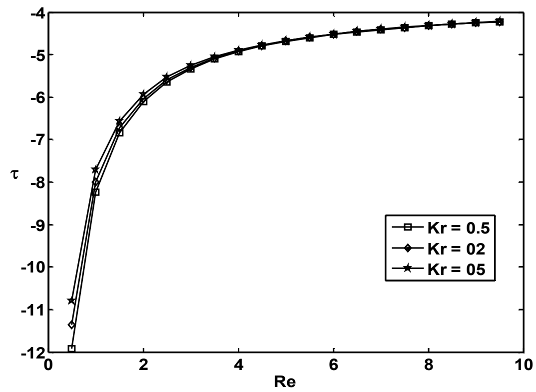

Fig. 9. Skin friction τ verses Re for ε = 0.01, So = 1, Sc = 0.22, M = 10.

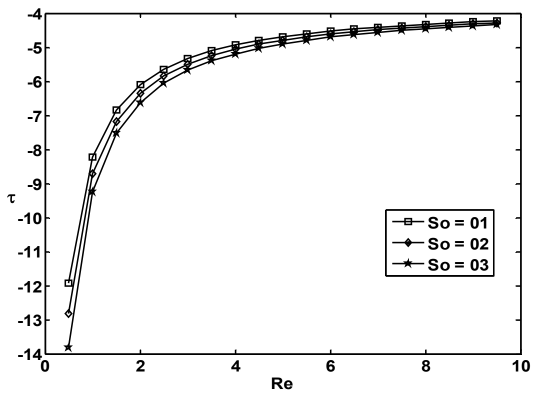

Fig. 10. Skin friction τ verses Re for Sc = 0.22, ε = 0.01, Kr = 0.5, M = 10.

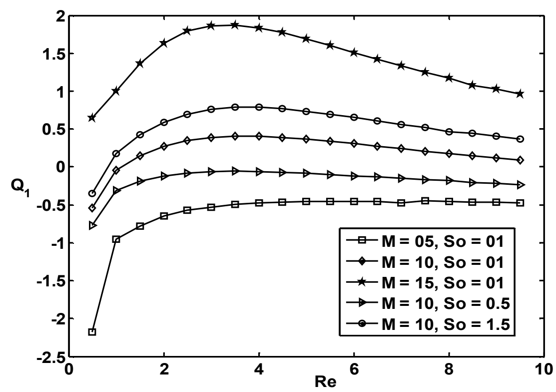

Fig. 11. The amplitude Q_1 of the first order skin friction verses Re for Sc = 0.22, ε = 0.01, Kr = 0.5.

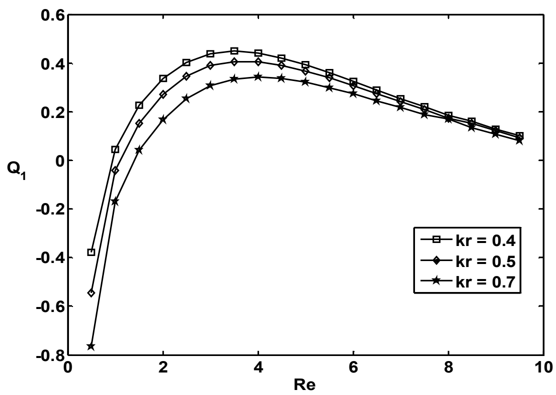

Fig. 12. The amplitude Q_1 of the first order skin friction verses **Re** for Sc = 0.22, ε = 0.01, So = 1, M = 10.

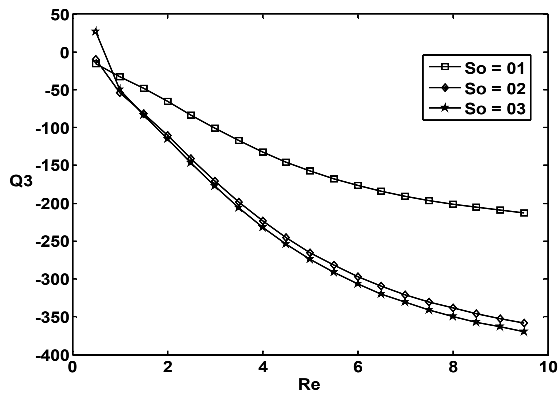

Fig. 15. The amplitude Q_3 of the first order Sherwood number verses Re for Sc = 0.22, M = 10, ε = 0.01, Kr = 0.5.

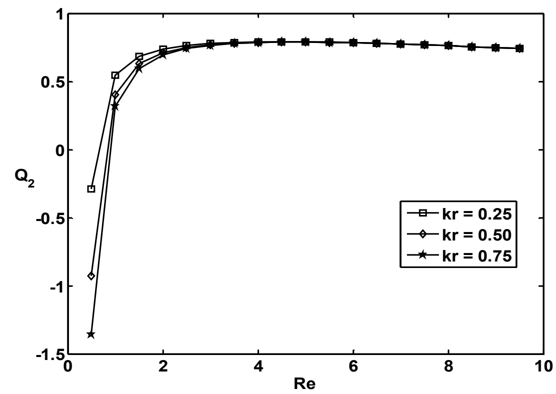

Fig. 13. The amplitude Q_2 of the first order Nusselt number verses Re for So = 1, M = 10, Sc = 0.22, ε = 0.01.

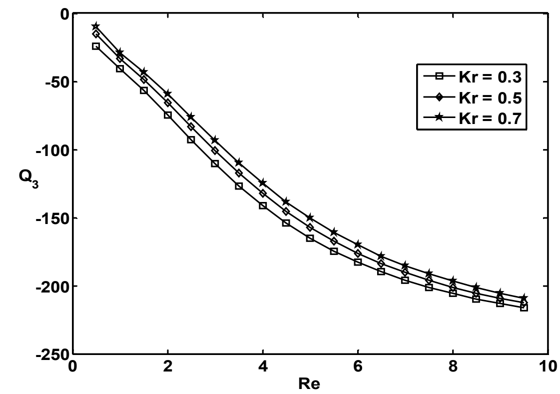

Fig. 16. The amplitude Q_3 of the first order Sherwood number verses Re for Sc = 0.22, So = 1, M =10, ε = 0.01.

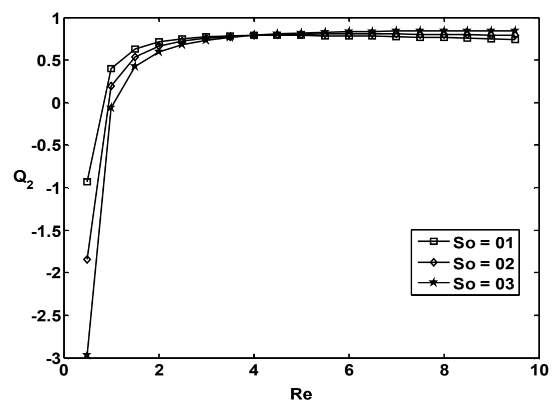

Fig. 14. The amplitude Q_2 of the first order Nusselt number verses Re for Sc = 0.22, ε = 0.01, Kr = 0.5, M = 10.

From Fig. 15 it is seen that, for Re > 1, the amplitude of the Sherwood number Q_3 decreases as the Soret number So increases. Fig. 16 shows that the magnitude of the Sherwood number $|Q_3|$ decreases as the chemical reaction parameter Kr increases.

5. Conclusions

The results obtained from our investigation lead to the following conclusions.

1. The velocity decreases with the increase in magnetic parameter and chemical reaction parameter.

2. The magnitude of the skin friction decreases with an increase in the chemical reaction parameter.
3. The magnitude of the skin friction increases with an increase in the thermo diffusion parameter.
4. An increase in magnetic parameter or Thermo diffusion parameter leads to increase in the amplitude of first order skin friction.
5. The amplitude of the first order skin friction as well as the magnitude of the amplitude of first order Sherwood number decreases with an increase in the chemical reaction parameter.

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Appendix

b = Re Sc; c = Sc Kr; d = Gr Re;

e = Gm Re; f = Sc So; $a = \bar{\lambda}$ Re ;

g = π^2 + MRe^2; $\lambda = \frac{1+\sqrt{1+4M}}{2}$;

$\overline{\lambda} = \dfrac{1-\sqrt{1+4M}}{2}$; $t_0 = \dfrac{b+\sqrt{b^2-4ac}}{2}$;

$t_1 = \Pr \mathrm{Re}$; $t_2 = \lambda \mathrm{Re}$; $t_3 = t_0 + t_1$;

$t_4 = t_2 + t_1$; $t_5 = t_0 + t_2$;

$t_6 = \dfrac{b+\sqrt{b^2-4c}}{2}$; $t_7 = \dfrac{t_1+\sqrt{t_1^2+4\pi^2}}{2}$;

$\xi = \dfrac{t_2+\sqrt{t_2^2+4\pi^2}}{2}$; $\overline{\xi} = \dfrac{a+\sqrt{a^2+4\pi^2}}{2}$;

$t_8 = \xi + t_1$; $t_9 = \overline{\xi} + t_1$; $t_{10} = \xi + t_0$;

$t_{11} = \overline{\xi} + t_0$; $t_{12} = \dfrac{b+\sqrt{b^2+4\pi^2-4c}}{2}$;

$t_{13} = \xi + t_2$; $t_{14} = \overline{\xi} + t_2$;

$t_{15} = \dfrac{\mathrm{Re}+\sqrt{\mathrm{Re}^2+4g}}{2}$; $t_{16} = \xi + 2t_0$;

$t_{17} = \xi + 2t_1$; $t_{18} = \xi + 2t_2$; $t_{19} = \xi + t_3$; $t_{20} = \xi + t_4$; $t_{21} = \xi + t_5$; $t_{22} = \overline{\xi} + 2t_0$; $t_{23} = \overline{\xi} + 2t_1$; $t_{24} = \overline{\xi} + 2t_2$; $t_{25} = \overline{\xi} + t_3$; $t_{26} = \overline{\xi} + t_4$; $t_{27} = \overline{\xi} + t_5$; $t_{28} = t_2 + t_7$; $t_{29} = t_2 + t_8$; $t_{30} = t_2 + t_9$; $t_{31} = t_2 + t_{10}$; $t_{32} = t_2 + t_{11}$; $t_{33} = t_2 + t_{12}$; $t_{34} = t_2 + t_{13}$; $t_{35} = t_2 + t_{14}$; $t_{36} = t_2 + t_{15}$; $t_{37} = t_7 + t_1$; $t_{38} = t_8 + t_1$; $t_{39} = t_9 + t_1$; $t_{40} = t_{10} + t_1$; $t_{41} = t_{11} + t_1$; $t_{42} = t_1 + t_{12}$; $t_{43} = t_{14} + t_1$; $t_{44} = t_{14} + t_1$; $t_{45} = t_{15} + t_1$; $t_{46} = t_0 + t_7$; $t_{47} = t_0 + t_8$; $t_{48} = t_0 + t_9$; $t_{49} = t_0 + t_{10}$; $t_{50} = t_0 + t_{11}$; $t_{51} = t_0 + t_{12}$; $t_{52} = t_0 + t_{13}$; $t_{53} = t_0 + t_{14}$; $t_{54} = t_0 + t_{15}$; $t_{55} = \xi + t_6$;

$t_{56} = \overline{\xi} + t_6$; $C_1 = \dfrac{-ScSot_1^2}{t_1^2 - t_1 b - c}$;

$C_0 = 1 - C_1$; $C_2 = -(C_3 + C_4 + U)$;

$C_3 = \dfrac{eC_1 + d}{M\,\mathrm{Re}^2 + \mathrm{Re}\,t_1 - t_1^2}$;

$C_4 = \dfrac{eC_0}{M\,\mathrm{Re}^2 + \mathrm{Re}\,t_0 - t_0^2}$;

$C_6 = \dfrac{C_4^2(t_0^2 \Pr + M\,\mathrm{Re}\,t_1)}{2t_0t_1 - 4t_0^2}$;

$C_7 = \dfrac{C_3^2(-t_1 \Pr - M\,\mathrm{Re})}{2t_1}$;

$C_8 = \dfrac{C_2^2(-t_2^2 \Pr + M\,\mathrm{Re}\,t_1)}{2t_1t_2 - t_2^2}$;

$C_9 = \dfrac{-2C_3C_4(t_0t_1 \Pr + M\,\mathrm{Re}\,t_1)}{t_0^2 + t_0t_1}$;

$C_{10} = \dfrac{-2C_2C_3(t_2t_1 \Pr + M\,\mathrm{Re}\,t_1)}{t_2^2 + t_1t_2}$;

$C_{11} = \dfrac{-2C_2C_4(t_0t_2 \Pr + M\,\mathrm{Re}\,t_1)}{(t_0+t_2)^2 - (t_0+t_2)t_1}$;

$C_5 = -(C_6 + C_7 + C_8 + C_9 + C_{10} + C_{11}$;

$C_{13} = \dfrac{-f\,C_5t_1^2}{t_1^2 - bt_1 + c}$; $C_{14} = \dfrac{-4f\,C_6t_0^2}{4t_0^2 - 2bt_0 + c}$;

$C_{15} = \dfrac{-4f\,C_7t_1^2}{4t_1^2 - 2bt_1 + c}$;

$C_{16} = \dfrac{-4f\,C_8t_2^2}{4t_2^2 - 2bt_2 + c}$;

$C_{17} = \dfrac{-f\,C_9t_3^2}{t_3^2 - bt_3 + c}$; $C_{18} = \dfrac{-f\,C_{10}t_4^2}{t_4^2 - bt_4 + c}$;

$C_{19} = \dfrac{-f\,C_{11}t_5^2}{t_5^2 - bt_5 + c}$;

$C_{21} = \dfrac{dC_5 + eC_{13}}{M\,\mathrm{Re}^2 + \mathrm{Re}\,t_1 - t_1^2}$;

$C_{22} = \dfrac{dC_6 + eC_{14}}{M\,\mathrm{Re}^2 + 2\,\mathrm{Re}\,t_0 - 4t_0^2}$;

$C_{23} = \dfrac{dC_7 + eC_{15}}{M\,\mathrm{Re}^2 + 2\,\mathrm{Re}\,t_1 - 4t_1^2}$;

$C_{24} = \dfrac{dC_8 + eC_{16}}{M\,\mathrm{Re}^2 + 2\,\mathrm{Re}\,t_2 - 4t_2^2}$;

$$C_{25}=\frac{dC_9+eC_{17}}{M\,\mathrm{Re}^2+\mathrm{Re}\,t_3-t_3{}^2};$$

$$C_{26}=\frac{dC_{10}+eC_{18}}{M\,\mathrm{Re}^2+\mathrm{Re}\,t_4-t_4{}^2};$$

$$C_{27}=\frac{dC_{11}+eC_{19}}{M\,\mathrm{Re}^2+\mathrm{Re}\,t_5-t_5{}^2};$$

$$C_{28}=\frac{eC_{12}}{M\,\mathrm{Re}^2+\mathrm{Re}\,t_6-t_6{}^2};$$

$$C_{12}=-(C_{13}+C_{14}+C_{15}+C_{16}+C_{17}+C_{18}+C_{19});$$

$$C_{20}=-(C_{21}+C_{22}+C_{23}+C_{24}+C_{25}+C_{26}+C_{27}+C_{28});$$

$$C_{30}=\frac{t_1{}^2\bar{\xi}}{(\bar{\xi}-\xi)(\xi^2+t_1\xi-\pi^2)};$$

$$C_{31}=\frac{-t_1{}^2\xi}{(\bar{\xi}-\xi)(\bar{\xi}^2+t_1\bar{\xi}-\pi^2)};$$

$$C_{29}=-(C_{30}+C_{31});$$

$$C_{33}=\frac{fC_{29}(\pi^2-t_7{}^2)}{t_7{}^2-bt_7+c-\pi^2};$$

$$C_{34}=\frac{fC_{30}(\pi^2-t_8{}^2)-\dfrac{bt_1C_1\bar{\xi}}{\xi-\bar{\xi}}}{t_8{}^2-bt_8+c-\pi^2};$$

$$C_{35}=\frac{fC_{31}(\pi^2-t_9{}^2)+\dfrac{bt_1C_1\xi}{\xi-\bar{\xi}}}{t_9{}^2-bt_9+c-\pi^2};$$

$$C_{36}=\frac{-bt_0C_0\bar{\xi}}{(\xi-\bar{\xi})(t_{10}{}^2-bt_{10}+c-\pi^2)};$$

$$C_{37}=\frac{-bt_0C_0\xi}{(\xi-\bar{\xi})(t_{11}{}^2-bt_{11}+c-\pi^2)};$$

$$C_{32}=-(C_{33}+C_{34}+C_{35}+C_{36}+C_{37});$$

$$C_{39}=\frac{eC_{33}+dC_{29}}{g+\mathrm{Re}\,t_7-t_7{}^2};$$

$$C_{40}=\frac{\dfrac{\mathrm{Re}\,t_1C_3\bar{\xi}}{\bar{\xi}-\xi}-dC_{30}-eC_{34}}{t_8{}^2-\mathrm{Re}\,t_8-g};$$

$$C_{41}=\frac{\dfrac{\mathrm{Re}\,t_1C_3\xi}{\xi-\bar{\xi}}-dC_{31}-eC_{35}}{t_9{}^2-\mathrm{Re}\,t_9-g};$$

$$C_{42}=\frac{\dfrac{\mathrm{Re}\,t_0C_4\bar{\xi}}{\bar{\xi}-\xi}-eC_{36}}{t_{10}{}^2-\mathrm{Re}\,t_{10}-g};$$

$$C_{43}=\frac{\dfrac{\mathrm{Re}\,t_0C_4\xi}{\xi-\bar{\xi}}-eC_{37}}{t_{11}{}^2-\mathrm{Re}\,t_{11}-g};$$

$$C_{44}=\frac{eC_{32}}{g+\mathrm{Re}\,t_{12}-t_{12}{}^2};$$

$$C_{45}=\frac{t_2C_2\,\mathrm{Re}\,\bar{\xi}}{(\bar{\xi}-\xi)(t_{13}{}^2-\mathrm{Re}\,t_{13}-g)};$$

$$C_{46}=\frac{t_2C_2\,\mathrm{Re}\,\xi}{(\xi-\bar{\xi})(t_{14}{}^2-\mathrm{Re}\,t_{14}-g)};$$

$$C_{38}=-(C_{39}+C_{40}+C_{41}+C_{42}+C_{43}+C_{44}+C_{45}+C_{46});$$

$$C_{48}=\frac{t_1{}^2C_5\bar{\xi}}{(\bar{\xi}-\xi)(t_8{}^2-t_1t_8-\pi^2)};$$

$$C_{49}=\frac{t_1{}^2C_5\xi}{(\bar{\xi}-\xi)(t_9{}^2-t_1t_9-\pi^2)};$$

$$C_{50}=\frac{2t_0t_1C_6\bar{\xi}}{(\bar{\xi}-\xi)(t_{16}{}^2-t_1t_{16}-\pi^2)};$$

$$C_{51}=\frac{2t_1{}^2C_7\bar{\xi}}{(\bar{\xi}-\xi)(t_{17}{}^2-t_1t_{17}-\pi^2)};$$

$$C_{52}=\frac{2t_2t_1C_8\bar{\xi}}{(\bar{\xi}-\xi)(t_{18}{}^2-t_1t_{18}-\pi^2)};$$

$$C_{53}=\frac{t_3t_1C_9\bar{\xi}}{(\bar{\xi}-\xi)(t_{19}{}^2-t_1t_{19}-\pi^2)};$$

$$C_{54}=\frac{t_4t_1C_{10}\bar{\xi}}{(\bar{\xi}-\xi)(t_{20}{}^2-t_1t_{20}-\pi^2)};$$

$$C_{55}=\frac{t_5t_1C_{11}\bar{\xi}}{(\bar{\xi}-\xi)(t_{21}{}^2-t_1t_{21}-\pi^2)};$$

$$C_{56}=\frac{2t_0t_1C_6\xi}{(\bar{\xi}-\xi)(t_{22}{}^2-t_1t_{22}-\pi^2)};$$

$$C_{57}=\frac{2t_1{}^2C_7\xi}{(\xi-\bar{\xi})(t_{23}{}^2-t_1t_{23}-\pi^2)};$$

$$C_{58}=\frac{2t_2t_1C_8\xi}{(\xi-\bar{\xi})(t_{24}{}^2-t_1t_{24}-\pi^2)};$$

$$C_{59}=\frac{t_3t_1C_9\xi}{(\xi-\bar{\xi})(t_{25}{}^2-t_1t_{25}-\pi^2)};$$

$$C_{60}=\frac{t_4t_1C_{10}\xi}{(\xi-\bar{\xi})(t_{26}{}^2-t_1t_{26}-\pi^2)};$$

$$C_{61}=\frac{t_5t_1C_{11}\xi}{(\xi-\bar{\xi})(t_{27}{}^2-t_1t_{27}-\pi^2)};$$

$$C_{62}=\frac{2\Pr t_2t_7C_2C_{39}+2M\operatorname{Re}t_1C_2C_{39}}{\pi^2+t_1t_{28}-t_{28}{}^2};$$

$$C_{63}=\frac{2\Pr t_2t_8C_2C_{40}+2M\operatorname{Re}t_1C_2C_{40}}{\pi^2+t_1t_{29}-t_{29}{}^2};$$

$$C_{64}=\frac{2\Pr t_2t_9C_2C_{41}+2M\operatorname{Re}t_1C_2C_{41}}{\pi^2+t_1t_{30}-t_{30}{}^2};$$

$$C_{65}=\frac{2\Pr t_2t_{10}C_2C_{42}+2M\operatorname{Re}t_1C_2C_{42}}{\pi^2+t_1t_{31}-t_{31}{}^2};$$

$$C_{66}=\frac{2\Pr t_2t_{11}C_2C_{43}+2M\operatorname{Re}t_1C_2C_{43}}{\pi^2+t_1t_{32}-t_{32}{}^2};$$

$$C_{67}=\frac{2\Pr t_2t_{12}C_2C_{44}+2M\operatorname{Re}t_1C_2C_{44}}{\pi^2+t_1t_{33}-t_{33}{}^2};$$

$$C_{68}=\frac{2\Pr t_2t_{13}C_2C_{45}+2M\operatorname{Re}t_1C_2C_{45}}{\pi^2+t_1t_{34}-t_{34}{}^2};$$

$$C_{69}=\frac{2\Pr t_2t_{14}C_2C_{46}+2M\operatorname{Re}t_1C_2C_{46}}{\pi^2+t_1t_{35}-t_{35}{}^2};$$

$$C_{70}=\frac{2\Pr t_2t_{15}C_2C_{38}+2M\operatorname{Re}t_1C_2C_{38}}{\pi^2+t_1t_{36}-t_{36}{}^2};$$

$$C_{71}=\frac{2\Pr t_1t_7C_3C_{39}+2M\operatorname{Re}t_1C_3C_{39}}{\pi^2+t_1t_{37}-t_{37}{}^2};$$

$$C_{72}=\frac{2\Pr t_1t_8C_3C_{40}+2M\operatorname{Re}t_1C_3C_{40}}{\pi^2+t_1t_{38}-t_{38}{}^2};$$

$$C_{73}=\frac{2\Pr t_1t_9C_3C_{41}+2M\operatorname{Re}t_1C_3C_{41}}{\pi^2+t_1t_{39}-t_{39}{}^2};$$

$$C_{74}=\frac{2\Pr t_1t_{10}C_3C_{42}+2M\operatorname{Re}t_1C_3C_{42}}{\pi^2+t_1t_{40}-t_{40}{}^2};$$

$$C_{75}=\frac{2\Pr t_1t_{11}C_3C_{43}+2M\operatorname{Re}t_1C_3C_{43}}{\pi^2+t_1t_{41}-t_{41}{}^2};$$

$$C_{76}=\frac{2\Pr t_1t_{12}C_3C_{44}+2M\operatorname{Re}t_1C_3C_{44}}{\pi^2+t_1t_{42}-t_{42}{}^2};$$

$$C_{77}=\frac{2\Pr t_1t_{13}C_3C_{45}+2M\operatorname{Re}t_1C_3C_{45}}{\pi^2+t_1t_{43}-t_{43}{}^2};$$

$$C_{78}=\frac{2\Pr t_1t_{14}C_3C_{46}+2M\operatorname{Re}t_1C_3C_{46}}{\pi^2+t_1t_{44}-t_{44}{}^2};$$

$$C_{79}=\frac{2\Pr t_1t_{15}C_3C_{38}+2M\operatorname{Re}t_1C_3C_{38}}{\pi^2+t_1t_{45}-t_{45}{}^2};$$

$$C_{80}=\frac{2\Pr t_0t_7C_4C_{39}+2M\operatorname{Re}t_1C_4C_{39}}{\pi^2+t_1t_{46}-t_{46}{}^2};$$

$$C_{81}=\frac{2\Pr t_0t_8C_4C_{40}+2M\operatorname{Re}t_1C_4C_{40}}{\pi^2+t_1t_{47}-t_{47}{}^2};$$

$$C_{82}=\frac{2\Pr t_0t_9C_4C_{41}+2M\operatorname{Re}t_1C_4C_{41}}{\pi^2+t_1t_{48}-t_{48}{}^2};$$

$$C_{83}=\frac{2\Pr t_0t_{10}C_4C_{42}+2M\operatorname{Re}t_1C_4C_{42}}{\pi^2+t_1t_{49}-t_{49}{}^2};$$

$$C_{84}=\frac{2\Pr t_{11}t_0C_4C_{43}+2M\operatorname{Re}t_1C_4C_{43}}{\pi^2+t_1t_{50}-t_{50}{}^2};$$

$$C_{85}=\frac{2\Pr t_0 t_{12} C_4 C_{44}+2M\operatorname{Re} t_1 C_4 C_{44}}{\pi^2+t_1 t_{51}-t_{51}^{\;2}};$$

$$C_{86}=\frac{2\Pr t_0 t_{13} C_4 C_{45}+2M\operatorname{Re} t_1 C_4 C_{45}}{\pi^2+t_1 t_{52}-t_{52}^{\;2}};$$

$$C_{87}=\frac{2\Pr t_0 t_{14} C_4 C_{46}+2M\operatorname{Re} t_1 C_4 C_{46}}{\pi^2+t_1 t_{53}-t_{53}^{\;2}};$$

$$C_{88}=\frac{2\Pr t_0 t_{15} C_4 C_{38}+2M\operatorname{Re} t_1 C_4 C_{38}}{\pi^2+t_1 t_{54}-t_{54}^{\;2}};$$

$$\begin{aligned}C_{47}=-(&C_{48}+C_{49}+C_{50}+C_{51}+C_{52}+C_{53}\\&+C_{54}+C_{55}+C_{56}+C_{57}+C_{58}+C_{59}+C_{60}\\&+C_{61}+C_{62}+C_{63}+C_{64}+C_{65}+C_{66}+C_{67}\\&+C_{68}+C_{69}+C_{70}+C_{71}+C_{72}+C_{73}+C_{74}\\&+C_{75}+C_{76}+C_{77}+C_{78}+C_{79}+C_{80}+C_{81}\\&+C_{82}+C_{83}+C_{84}+C_{85}+C_{86}+C_{87}+C_{88});\end{aligned}$$

$$C_{90}=\frac{bt_6 C_{12}\bar{\xi}}{(\bar{\xi}-\xi)(t_{55}^{\;2}-bt_{55}+c-\pi^2)};$$

$$C_{91}=\frac{bt_6 C_{12}\xi}{(\xi-\bar{\xi})(t_{56}^{\;2}-bt_{56}+c-\pi^2)};$$

$$C_{92}=\frac{f\,C_{47}(\pi^2-t_7^{\;2})}{t_7^{\;2}-bt_7+c-\pi^2};$$

$$C_{93}=\frac{\frac{bt_1 C_{13}\bar{\xi}}{\bar{\xi}-\xi}+f\,C_{48}(\pi^2-t_8^{\;2})}{t_8^{\;2}-bt_8+c-\pi^2};$$

$$C_{94}=\frac{\frac{bt_1 C_{13}\xi}{\xi-\bar{\xi}}+f\,C_{49}(\pi^2-t_9^{\;2})}{t_9^{\;2}-bt_9+c-\pi^2};$$

$$C_{95}=\frac{\frac{2bt_0 C_{14}\bar{\xi}}{\bar{\xi}-\xi}+f\,C_{50}(\pi^2-t_{16}^{\;2})}{t_{16}^{\;2}-bt_{16}+c-\pi^2};$$

$$C_{96}=\frac{\frac{2bt_1 C_{15}\bar{\xi}}{\bar{\xi}-\xi}+f\,C_{51}(\pi^2-t_{17}^{\;2})}{t_{17}^{\;2}-bt_{17}+c-\pi^2};$$

$$C_{97}=\frac{\frac{2bt_2 C_{16}\bar{\xi}}{\bar{\xi}-\xi}+f\,C_{52}(\pi^2-t_{18}^{\;2})}{t_{18}^{\;2}-bt_{18}+c-\pi^2};$$

$$C_{98}=\frac{\frac{bt_3 C_{17}\bar{\xi}}{\bar{\xi}-\xi}+f\,C_{53}(\pi^2-t_{19}^{\;2})}{t_{19}^{\;2}-bt_{19}+c-\pi^2};$$

$$C_{99}=\frac{\frac{bt_4 C_{18}\bar{\xi}}{\bar{\xi}-\xi}+f\,C_{54}(\pi^2-t_{20}^{\;2})}{t_{20}^{\;2}-bt_{20}+c-\pi^2};$$

$$C_{100}=\frac{\frac{bt_5 C_{19}\bar{\xi}}{\xi-\bar{\xi}}+f\,C_{55}(\pi^2-t_{21}^{\;2})}{t_{21}^{\;2}-bt_{21}+c-\pi^2};$$

$$C_{101}=\frac{\frac{2bt_0 C_4\xi}{\xi-\bar{\xi}}+f\,C_{56}(\pi^2-t_{22}^{\;2})}{t_{22}^{\;2}-bt_{22}+c-\pi^2};$$

$$C_{102}=\frac{\frac{2bt_1 C_{15}\xi}{\xi-\bar{\xi}}+f\,C_{57}(\pi^2-t_{23}^{\;2})}{t_{23}^{\;2}-bt_{23}+c-\pi^2};$$

$$C_{103}=\frac{\frac{2bt_2 C_{16}\xi}{\xi-\bar{\xi}}+f\,C_{58}(\pi^2-t_{24}^{\;2})}{t_{24}^{\;2}-bt_{24}+c-\pi^2};$$

$$C_{104}=\frac{\frac{bt_3 C_{17}\xi}{\xi-\bar{\xi}}+f\,C_{59}(\pi^2-t_{25}^{\;2})}{t_{25}^{\;2}-bt_{25}+c-\pi^2};$$

$$C_{105}=\frac{\frac{bt_4 C_{18}\xi}{\xi-\bar{\xi}}+f\,C_{60}(\pi^2-t_{26}^{\;2})}{t_{26}^{\;2}-bt_{26}+c-\pi^2};$$

$$C_{106} = \frac{\dfrac{bt_5 C_{19}\xi}{\xi - \bar{\xi}} + f\,C_{61}(\pi^2 - t_{27}{}^2)}{t_{27}{}^2 - bt_{27} + c - \pi^2};$$

$$C_{107} = \frac{f\,C_{62}(\pi^2 - t_{28}{}^2)}{t_{28}{}^2 - bt_{28} + c - \pi^2};$$

$$C_{108} = \frac{f\,C_{63}(\pi^2 - t_{29}{}^2)}{t_{29}{}^2 - bt_{29} + c - \pi^2};$$

$$C_{109} = \frac{f\,C_{64}(\pi^2 - t_{30}{}^2)}{t_{30}{}^2 - bt_{30} + c - \pi^2};$$

$$C_{110} = \frac{f\,C_{65}(\pi^2 - t_{31}{}^2)}{t_{31}{}^2 - bt_{31} + c - \pi^2};$$

$$C_{111} = \frac{f\,C_{66}(\pi^2 - t_{32}{}^2)}{t_{32}{}^2 - bt_{32} + c - \pi^2};$$

$$C_{112} = \frac{f\,C_{67}(\pi^2 - t_{33}{}^2)}{t_{33}{}^2 - bt_{33} + c - \pi^2};$$

$$C_{113} = \frac{f\,C_{68}(\pi^2 - t_{34}{}^2)}{t_{34}{}^2 - bt_{34} + c - \pi^2};$$

$$C_{114} = \frac{f\,C_{69}(\pi^2 - t_{35}{}^2)}{t_{35}{}^2 - bt_{35} + c - \pi^2};$$

$$C_{115} = \frac{f\,C_{70}(\pi^2 - t_{36}{}^2)}{t_{36}{}^2 - bt_{36} + c - \pi^2};$$

$$C_{116} = \frac{f\,C_{71}(\pi^2 - t_{37}{}^2)}{t_{37}{}^2 - bt_{37} + c - \pi^2};$$

$$C_{117} = \frac{f\,C_{72}(\pi^2 - t_{38}{}^2)}{t_{38}{}^2 - bt_{38} + c - \pi^2};$$

$$C_{118} = \frac{f\,C_{73}(\pi^2 - t_{39}{}^2)}{t_{39}{}^2 - bt_{39} + c - \pi^2};$$

$$C_{119} = \frac{f\,C_{74}(\pi^2 - t_{40}{}^2)}{t_{40}{}^2 - bt_{40} + c - \pi^2};$$

$$C_{120} = \frac{f\,C_{75}(\pi^2 - t_{41}{}^2)}{t_{41}{}^2 - bt_{41} + c - \pi^2};$$

$$C_{121} = \frac{f\,C_{76}(\pi^2 - t_{42}{}^2)}{t_{42}{}^2 - bt_{42} + c - \pi^2};$$

$$C_{122} = \frac{f\,C_{77}(\pi^2 - t_{43}{}^2)}{t_{43}{}^2 - bt_{43} + c - \pi^2};$$

$$C_{123} = \frac{f\,C_{78}(\pi^2 - t_{44}{}^2)}{t_{44}{}^2 - bt_{44} + c - \pi^2};$$

$$C_{124} = \frac{f\,C_{79}(\pi^2 - t_{45}{}^2)}{t_{45}{}^2 - bt_{45} + c - \pi^2};$$

$$C_{125} = \frac{f\,C_{80}(\pi^2 - t_{46}{}^2)}{t_{46}{}^2 - bt_{46} + c - \pi^2};$$

$$C_{126} = \frac{f\,C_{81}(\pi^2 - t_{47}{}^2)}{t_{47}{}^2 - bt_{47} + c - \pi^2};$$

$$C_{127} = \frac{f\,C_{82}(\pi^2 - t_{48}{}^2)}{t_{48}{}^2 - bt_{48} + c - \pi^2};$$

$$C_{128} = \frac{f\,C_{83}(\pi^2 - t_{49}{}^2)}{t_{49}{}^2 - bt_{49} + c - \pi^2};$$

$$C_{129} = \frac{f\,C_{84}(\pi^2 - t_{50}{}^2)}{t_{50}{}^2 - bt_{50} + c - \pi^2};$$

$$C_{130} = \frac{f\,C_{85}(\pi^2 - t_{51}{}^2)}{t_{51}{}^2 - bt_{51} + c - \pi^2};$$

$$C_{131} = \frac{f\,C_{86}(\pi^2 - t_{52}{}^2)}{t_{52}{}^2 - bt_{52} + c - \pi^2};$$

$$C_{132} = \frac{f\,C_{87}(\pi^2 - t_{53}{}^2)}{t_{53}{}^2 - bt_{53} + c - \pi^2};$$

$$C_{133} = \frac{f\,C_{88}(\pi^2 - t_{54}{}^2)}{t_{54}{}^2 - bt_{54} + c - \pi^2};$$

$$C_{89} = -(C_{90} + C_{91} + C_{92} + C_{93} + C_{94} + C_{95} + C_{96} + C_{97} + C_{98} + C_{99} + C_{100} + C_{101} + C_{102} + C_{103} + C_{104} + C_{105} + C_{106} + C_{107} + C_{108} + C_{109} + C_{110} + C_{111} + C_{112} + C_{113} + C_{114} + C_{115} + C_{116} + C_{117} + C_{118} + C_{119} + C_{120} + C_{121} + C_{122} + C_{123} + C_{124} + C_{125} + C_{126} + C_{127} + C_{128} + C_{129} + C_{130} + C_{131} + C_{132} + C_{133});$$

$$C_{135} = \frac{dC_{47} + eC_{92}}{g + \mathrm{Re}\, t_7 - t_7^{\,2}};$$

$$C_{136} = \frac{\dfrac{\mathrm{Re}\, t_1 C_{21} \bar{\xi}}{\bar{\xi} - \xi} - dC_{48} - eC_{93}}{t_8^{\,2} - \mathrm{Re}\, t_8 - g};$$

$$C_{137} = \frac{\dfrac{\mathrm{Re}\, t_1 C_{21} \xi}{\xi - \bar{\xi}} - dC_{49} - eC_{94}}{t_9^{\,2} - \mathrm{Re}\, t_9 - g};$$

$$C_{138} = \frac{\mathrm{Re}\, t_2 C_{20} \bar{\xi}}{(\bar{\xi} - \xi)(t_{13}^{\,2} - \mathrm{Re}\, t_{13} - g)};$$

$$C_{139} = \frac{\mathrm{Re}\, t_2 C_{20} \xi}{(\xi - \bar{\xi})(t_{14}^{\,2} - \mathrm{Re}\, t_{14} - g)};$$

$$C_{140} = \frac{\dfrac{2\,\mathrm{Re}\, t_0 C_{22} \bar{\xi}}{\bar{\xi} - \xi} - dC_{50} - eC_{95}}{t_{16}^{\,2} - \mathrm{Re}\, t_{16} - g};$$

$$C_{141} = \frac{\dfrac{2\,\mathrm{Re}\, t_1 C_{23} \bar{\xi}}{\bar{\xi} - \xi} - dC_{51} - eC_{96}}{t_{17}^{\,2} - \mathrm{Re}\, t_{17} - g};$$

$$C_{142} = \frac{\dfrac{\mathrm{Re}\, t_2 C_{24} \bar{\xi}}{\bar{\xi} - \xi} - dC_{52} - eC_{97}}{t_{18}^{\,2} - \mathrm{Re}\, t_{18} - g};$$

$$C_{143} = \frac{\dfrac{\mathrm{Re}\, t_3 C_{25} \bar{\xi}}{\bar{\xi} - \xi} - dC_{53} - eC_{98}}{t_{19}^{\,2} - \mathrm{Re}\, t_{19} - g};$$

$$C_{144} = \frac{\dfrac{\mathrm{Re}\, t_4 C_{26} \bar{\xi}}{\bar{\xi} - \xi} - dC_{54} - eC_{99}}{t_{20}^{\,2} - \mathrm{Re}\, t_{20} - g};$$

$$C_{145} = \frac{\dfrac{\mathrm{Re}\, t_5 C_{27} \bar{\xi}}{\bar{\xi} - \xi} - dC_{55} - eC_{100}}{t_{21}^{\,2} - \mathrm{Re}\, t_{21} - g};$$

$$C_{146} = \frac{\dfrac{2\,\mathrm{Re}\, t_0 C_{22} \xi}{\xi - \bar{\xi}} - dC_{56} - eC_{101}}{t_{22}^{\,2} - \mathrm{Re}\, t_{22} - g};$$

$$C_{147} = \frac{\dfrac{2\,\mathrm{Re}\, t_1 C_{23} \xi}{\xi - \bar{\xi}} - dC_{57} - eC_{102}}{t_{23}^{\,2} - \mathrm{Re}\, t_{23} - g};$$

$$C_{148} = \frac{\dfrac{2\,\mathrm{Re}\, t_2 C_{24} \xi}{\xi - \bar{\xi}} - dC_{58} - eC_{103}}{t_{24}^{\,2} - \mathrm{Re}\, t_{24} - g};$$

$$C_{149} = \frac{\dfrac{\mathrm{Re}\, t_3 C_{25} \xi}{\xi - \bar{\xi}} - dC_{59} - eC_{104}}{t_{25}^{\,2} - \mathrm{Re}\, t_{25} - g};$$

$$C_{150} = \frac{\dfrac{\mathrm{Re}\, t_4 C_{26} \xi}{\xi - \bar{\xi}} - dC_{60} - eC_{105}}{t_{26}^{\,2} - \mathrm{Re}\, t_{26} - g};$$

$$C_{151} = \frac{\dfrac{\mathrm{Re}\, t_5 C_{27} \xi}{\xi - \bar{\xi}} - dC_{61} - eC_{106}}{t_{27}^{\,2} - \mathrm{Re}\, t_{27} - g};$$

$$C_{152} = \frac{dC_{62} + eC_{107}}{g + \mathrm{Re}\, t_{28} - t_{28}^{\,2}};$$

$$C_{153} = \frac{dC_{63} + eC_{108}}{g + \mathrm{Re}\, t_{29} - t_{29}^{\,2}};$$

$$C_{154} = \frac{dC_{64} + eC_{109}}{g + \mathrm{Re}\, t_{30} - t_{30}^{\,2}};$$

$$C_{155} = \frac{dC_{65} + eC_{110}}{g + \mathrm{Re}\, t_{31} - t_{31}^{\,2}};$$

$$C_{156} = \frac{dC_{66} + eC_{111}}{g + \mathrm{Re}\, t_{32} - t_{32}^{\,2}};$$

$$C_{157} = \frac{dC_{67} + eC_{112}}{g + \mathrm{Re}\, t_{33} - t_{33}^{\,2}};$$

$$C_{158} = \frac{dC_{68} + eC_{113}}{g + \mathrm{Re}\, t_{34} - t_{34}^{\,2}};$$

$$C_{159} = \frac{dC_{69} + eC_{114}}{g + \mathrm{Re}\, t_{35} - t_{35}^{2}};$$

$$C_{160} = \frac{dC_{70} + eC_{115}}{g + \mathrm{Re}\, t_{36} - t_{36}^{2}};$$

$$C_{161} = \frac{dC_{71} + eC_{116}}{g + \mathrm{Re}\, t_{37} - t_{37}^{2}};$$

$$C_{162} = \frac{dC_{72} + eC_{117}}{g + \mathrm{Re}\, t_{38} - t_{38}^{2}};$$

$$C_{163} = \frac{dC_{73} + eC_{118}}{g + \mathrm{Re}\, t_{39} - t_{39}^{2}};$$

$$C_{164} = \frac{dC_{74} + eC_{119}}{g + \mathrm{Re}\, t_{40} - t_{40}^{2}};$$

$$C_{165} = \frac{dC_{75} + eC_{120}}{g + \mathrm{Re}\, t_{41} - t_{41}^{2}};$$

$$C_{166} = \frac{dC_{76} + eC_{121}}{g + \mathrm{Re}\, t_{42} - t_{42}^{2}};$$

$$C_{167} = \frac{dC_{77} + eC_{122}}{g + \mathrm{Re}\, t_{43} - t_{43}^{2}};$$

$$C_{168} = \frac{dC_{78} + eC_{123}}{g + \mathrm{Re}\, t_{44} - t_{44}^{2}};$$

$$C_{169} = \frac{dC_{79} + eC_{124}}{g + \mathrm{Re}\, t_{45} - t_{45}^{2}};$$

$$C_{170} = \frac{dC_{80} + eC_{125}}{g + \mathrm{Re}\, t_{46} - t_{46}^{2}};$$

$$C_{171} = \frac{dC_{81} + eC_{126}}{g + \mathrm{Re}\, t_{47} - t_{47}^{2}};$$

$$C_{172} = \frac{dC_{82} + eC_{127}}{g + \mathrm{Re}\, t_{48} - t_{48}^{2}};$$

$$C_{173} = \frac{dC_{83} + eC_{128}}{g + \mathrm{Re}\, t_{49} - t_{49}^{2}};$$

$$C_{174} = \frac{dC_{84} + eC_{129}}{g + \mathrm{Re}\, t_{50} - t_{50}^{2}};$$

$$C_{175} = \frac{dC_{85} + eC_{130}}{g + \mathrm{Re}\, t_{51} - t_{51}^{2}};$$

$$C_{176} = \frac{dC_{86} + eC_{131}}{g + \mathrm{Re}\, t_{52} - t_{52}^{2}};$$

$$C_{177} = \frac{dC_{87} + eC_{132}}{g + \mathrm{Re}\, t_{53} - t_{53}^{2}};$$

$$C_{178} = \frac{dC_{88} + eC_{133}}{g + \mathrm{Re}\, t_{54} - t_{54}^{2}};$$

$$C_{179} = \frac{\mathrm{Re}\, t_{6} C_{28} \bar{\xi}}{(\bar{\xi} - \xi)(t_{55}^{2} - \mathrm{Re}\, t_{55} - g)};$$

$$C_{180} = \frac{\mathrm{Re}\, t_{6} C_{28} \xi}{(\xi - \bar{\xi})(t_{56}^{2} - \mathrm{Re}\, t_{56} - g)};$$

$$\begin{aligned} C_{134} = -(&C_{135} + C_{136} + C_{137} + C_{138} + C_{139} \\ &+ C_{140} + C_{141} + C_{142} + C_{143} + C_{144} + C_{145} \\ &+ C_{146} + C_{147} + C_{148} + C_{149} + C_{150} + C_{151} \\ &+ C_{152} + C_{153} + C_{154} + C_{155} + C_{156} + C_{157} \\ &+ C_{158} + C_{159} + C_{160} + C_{161} + C_{162} + C_{163} \\ &+ C_{164} + C_{165} + C_{166} + C_{167} + C_{168} + C_{169} \\ &+ C_{170} + C_{171} + C_{172} + C_{173} + C_{174} + C_{175} \\ &+ C_{176} + C_{177} + C_{178} + C_{179} + C_{180}) \end{aligned}$$